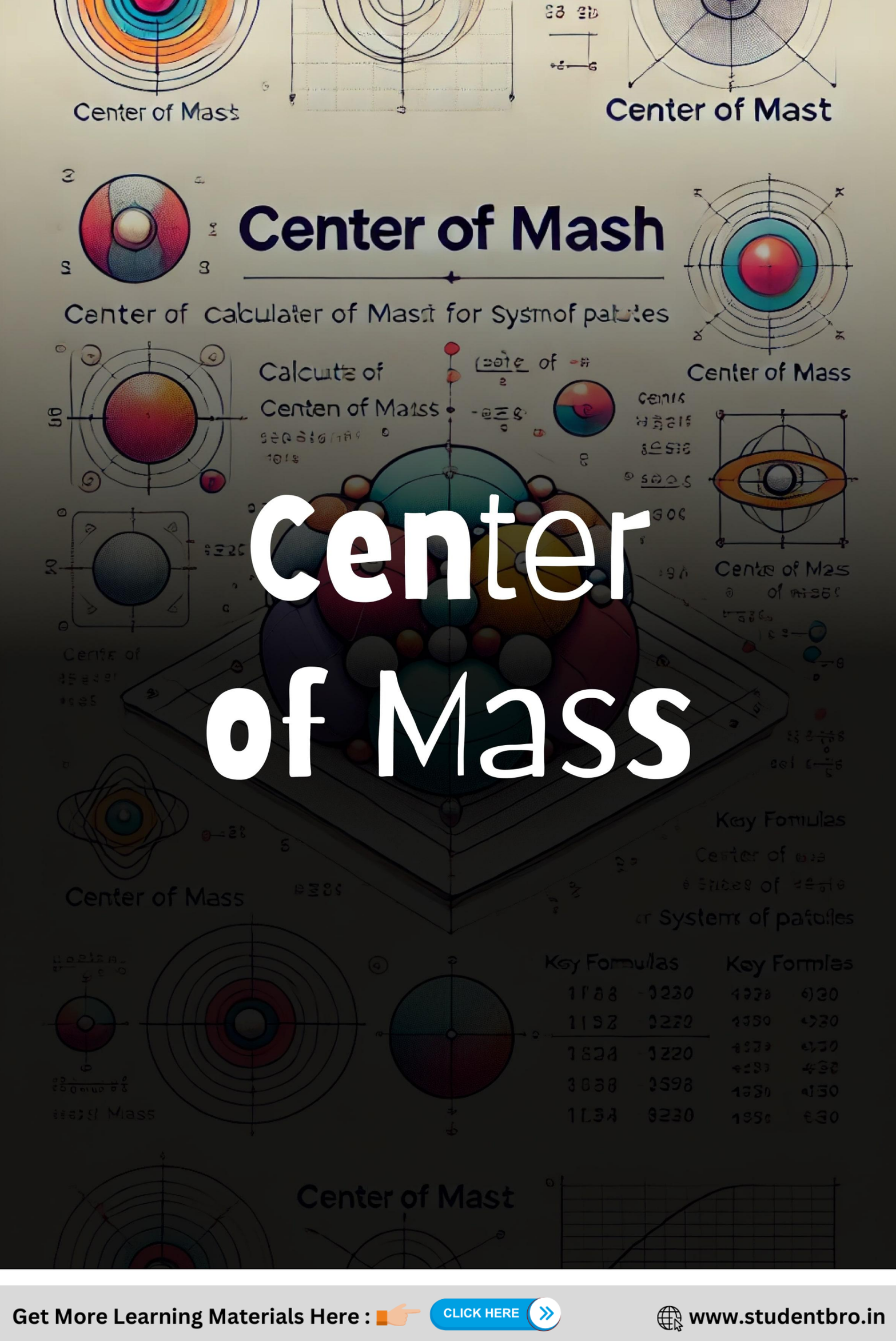


Center of Mass

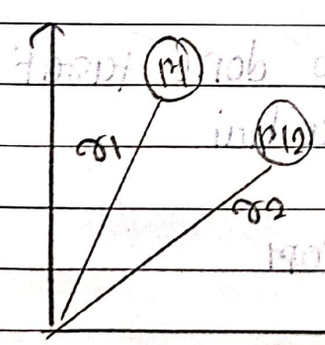
Center of Mass

→ A point where whole mass of the system can be Assume there.
→ It can be inside / outside the body
→ It Always on axis of symmetry
→ center of mass at a point where two axis of symmetry cut each other
→ center of mass near to havier object
→ position of com depends upon frame of reference or choice of coordinate
→ For Smaller object C.O.M and C.O.G act at Some point

↓
Where net gravitational force act

→ For large object C.O.M and C.O.G doesn't coincide. Slightly shift below the C.O.M.

* Discrete Mass System



$$\vec{r}_{com} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}$$

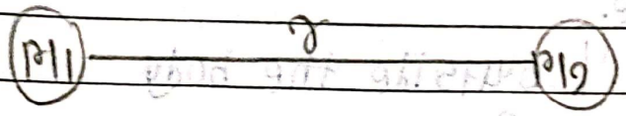
$$\vec{x}_{com} = \frac{m_1\vec{x}_1 + m_2\vec{x}_2}{m_1+m_2}$$

$$\vec{y}_{com} = \frac{m_1\vec{y}_1 + m_2\vec{y}_2}{m_1+m_2}$$

$$\vec{v}_{com} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}$$

$$\vec{a}_{com} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1+m_2}$$

* Find location of COM of two body system



r_1 = Position of COM from $m_1 = \frac{m_2 r}{m_1 + m_2}$

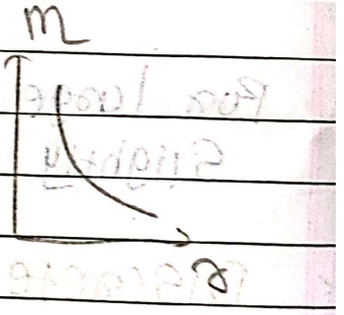
r_2 = Position of COM from $m_2 = \frac{m_1 r}{m_1 + m_2}$

$$\frac{(r_{cm})_1}{(r_{cm})_2} = \frac{m_2}{m_1}$$

$$m_1 r_{cm_1} = m_2 r_{cm_2}$$

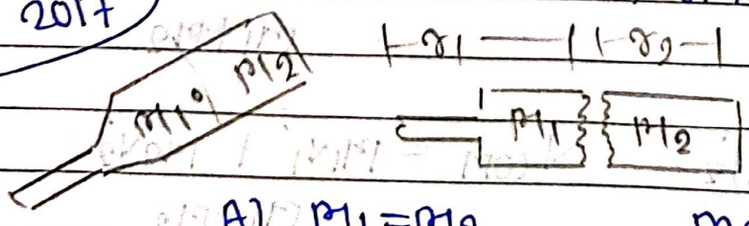
Moment of mass $\leftarrow mr$ = constant

$r \propto \frac{1}{m}$



COM ko point hota hai jisko dono taraf Moment of mass Barabar hota hai

Q) AIIMS 2017 But is broken from COM



A) $m_1 = m_2$

B) $m_1 > m_2$

C) $m_2 > m_1$

mr = constant

$r_1 > r_2$

$m_1 < m_2$

Q2) A point object of mass m is kept at (a,0) along x axis. What mass should be kept at (-3a,0) so that com lies at the origin

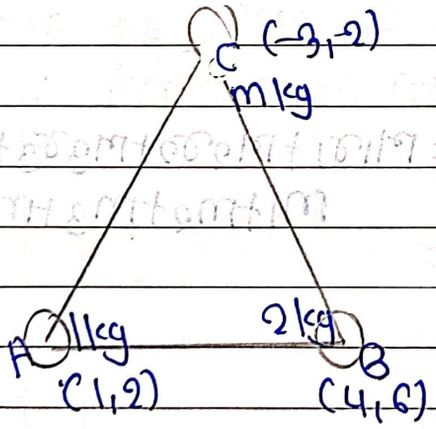
$$r_{com} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

$$m_1 r_1 + m_2 r_2 = 0$$

$$ma - 3am' = 0$$

$$m' = m/3$$

Q3) The coordinate of triangle ABC are A(1,2), B(4,6) C(-3,-2). Three Particle of mass 1kg, 2kg and m kg are placed the vertices of the triangle. If the coordinate of com are (3/5, 2), calculate m



$$x_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$\frac{3}{5} = \frac{(1\times1) + (2+4) + (-3+m)}{3+m}$$

$$9 + 3m = 45 - 15m$$

$$18m = 36$$

$$m = 2kg$$

Q4) Two bodies of mass 1kg & 2kg r lying in xy plane at (-1,2) and (2,4) respectively. What r the coordinate of com

$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(1)(-1) + (2)(2)}{3} = \frac{4-1}{3} = \frac{3}{3} = 1\,cm$$

$$y_{com} = \frac{m_1y_1 + m_2y_2}{m_1+m_2} = \frac{(1)(2)+(2)(4)}{3} = \frac{10}{3}\hat{j}$$

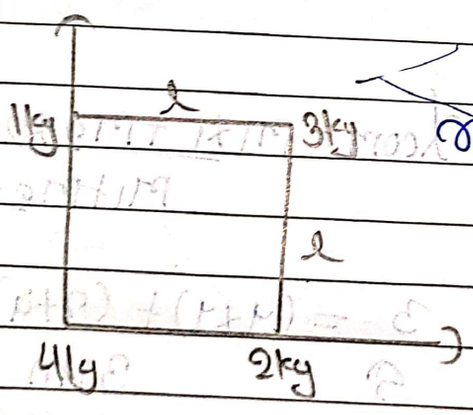
By Sir $\vec{r}_{com} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}$

$$= \frac{(1)(-\hat{i}+2\hat{j}) + 2(2\hat{i}+4\hat{j})}{3}$$

$$= \frac{-\hat{i}+2\hat{j}+4\hat{i}+8\hat{j}}{3}$$

$$r_{com} = 1, \frac{10}{3}$$

Q.2) Find location of com from origin



$$r_{com} = \frac{m_1r_1 + m_2r_2 + m_3r_3 + m_4r_4}{m_1+m_2+m_3+m_4}$$

$$r_{com} = \frac{0 + 2[l\hat{i}] + 3[\hat{i}+\hat{j}] + 4[\hat{j}]}{10}$$

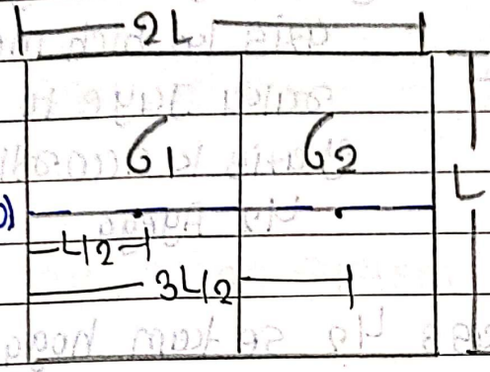
$$= \frac{2\hat{i}+3\hat{i}+3\hat{j}+4\hat{j}}{10}$$

$$= \frac{[5\hat{i}+7\hat{j}]l}{10}$$

$$r_{com} = \frac{l\hat{i}}{2} + \frac{7l\hat{j}}{10} \qquad r_{com} = \left(\frac{1}{2}, \frac{7}{10}\right)$$

Q7) III 2006

Find location of COM of system



$$x_{COM} = \frac{m_1 \frac{L}{2} + m_2 \frac{3L}{2}}{m_1 + m_2}$$

$$= \frac{\sigma_1 \frac{L^3}{2} + \sigma_2 \frac{3L^3}{2}}{\sigma_1 L^2 + \sigma_2 L^2}$$

$$= \left(\frac{\sigma_1 + 3\sigma_2}{\sigma_1 + \sigma_2}\right)\frac{L}{2}$$

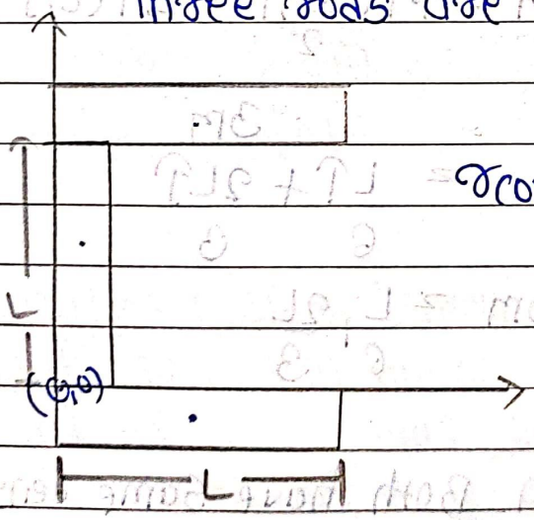
$m_1 = \sigma_1 L^2$

$m_2 = \sigma_2 L^2$

MR* If $\sigma_2 = 0$, then $r_{COM} = L/2$

If $\sigma_1 = \sigma_2$ then $r_{COM} = L$

Q8) Locate the COM of arrangement shown in the figure. Three rods are identical in length and mass

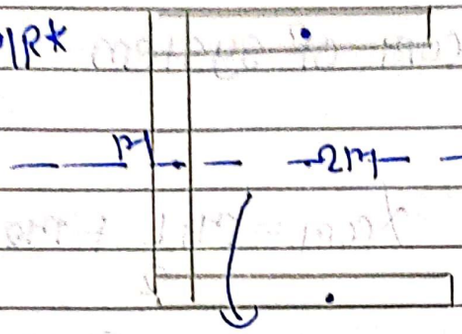


$$r_{COM} = \frac{m\frac{L}{2}\hat{\imath} + m\frac{L}{2}\hat{\jmath} + m\left[\frac{L}{2}\hat{\imath} + L\hat{\jmath}\right]}{3}$$

$$= \frac{mL\hat{\imath} + \frac{3Lm}{2}\hat{\jmath}}{3m}$$

$$r_{COM} = \frac{L}{3}\hat{\imath} + \frac{L}{2}\hat{\jmath}$$

$$r_{COM} = \left(\frac{L}{3}, \frac{L}{2}\right)$$

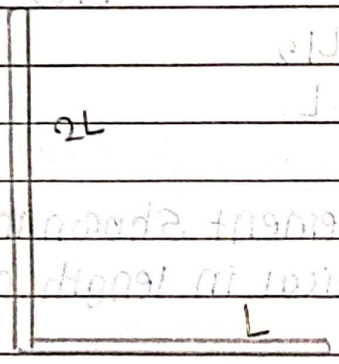


MRK

Agar symmetry axis ko bich me rakha jaye to y axis k according L/2 ayega

iska center of mass L/2 se kam hoga

Q9) Figure shows a composite system of two uniform rod of length as indicated. Then find com



$L \to M$

$2L \to 2M$

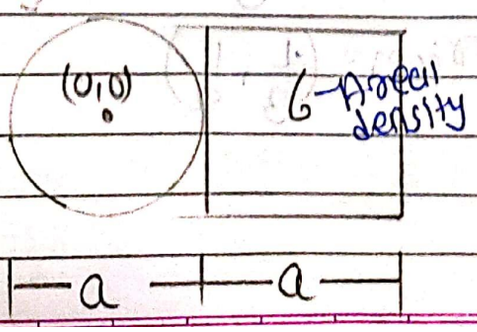
$$\vec{r}_{com} = \frac{M\left(\frac{L}{2}\right)\hat{i} + 2M(L)\hat{j}}{3M}$$

$$= \frac{L\hat{i}}{6} + \frac{2L\hat{j}}{3}$$

$$r_{com} = \frac{L}{6}, \frac{2L}{3}$$

Also use MRK

Q10) Find com from origin. Both have same density



m_1 circular plate $= \frac{6\pi a^2}{4}$

m_2 square plate $= 6a^2$

$COM = \frac{m_1x_1 + m_2x_2}{m_T}$

$= 0 + \frac{a \cdot 6a^2}{6a^2 + \frac{\pi 6a^2}{4}}$

$COM = \frac{4a}{4+\pi}$

* Continous mass system

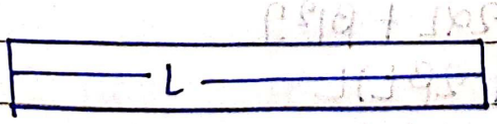
Linear mass density $\lambda = \frac{dm}{dL} = \frac{m}{L}$

Area mass density $\sigma = \frac{dm}{dA} = \frac{m}{A}$

Volume mass density $\rho = \frac{dm}{dV} = \frac{m}{V}$

$$x_{com} = \frac{\int x \cdot dm}{\int dm} \qquad r_{com} = \frac{\int r \cdot dm}{\int dm}$$

Q1) A rod of uniform thickness is placed along x axis with one end at the origin. If length of the rod is L and its linear mass density is proportional to x. then find dist. of com from origin.



$$x_{com} = \frac{\int x \cdot dm}{\int dm} = \frac{\int x \cdot \lambda \cdot dx}{\int \lambda \cdot dx}$$

$$= \frac{\int x^2 \cdot dx}{\int x \cdot dx}$$

$$= \frac{x^3}{3} \times \frac{2}{x^2}$$

$$x_{com} = \frac{2x}{3} = \frac{2L}{3}$$

Q2) $\lambda = \lambda_0 x^2$. then find com of rod about one end

$$x_{com} = \frac{\int x \cdot dm}{\int dm} = \frac{\int x \cdot \lambda \cdot dx}{\int \lambda \cdot dx} = \frac{\int x^3 \cdot dx}{\int x^2 dx}$$

$$= \frac{\frac{x^4}{4} \times 3}{x^3}$$

$$x_{com} = \frac{3x}{4} = \frac{3L}{4}$$

Q3) The linear mass density of a rod is of length (L) is kept along x axis varies as $\lambda = \alpha + \beta x$, where α and β are constant. The com of rod is

$$com = \frac{\int dm \cdot x}{\int dm} = \frac{\int \lambda \cdot dx \cdot x}{\int \lambda \cdot dx}$$

$$= \frac{\int (\alpha + \beta x) x \cdot dx}{\int (\alpha + \beta x) dx} = \frac{\int (\alpha x + \beta x^2) dx}{\int (\alpha + \beta x) dx}$$

$$= \frac{\alpha x^2 + \beta x^3}{2} \Big/ \frac{3}{\alpha x + \frac{\beta x^2}{2}}$$

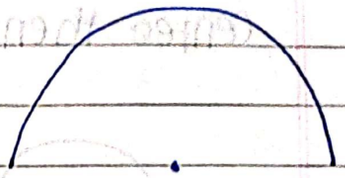
$$= \frac{[3\alpha L^2 + 2\beta L^3]}{6[2\alpha L + \beta L^2]} 2$$

$$= \frac{[3\alpha L^2 + 2\beta L^3]}{3[2\alpha L + \beta L^2]}$$

$$= \frac{(3\alpha + 2\beta L)L}{3(2\alpha + \beta L)}$$

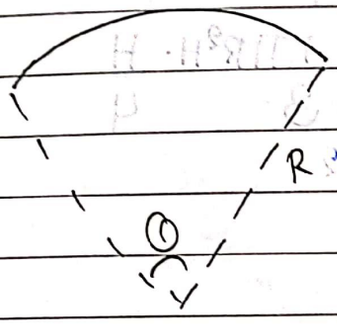
MR* If
$\lambda = \alpha + \beta x$
$x = 0$
$\lambda = \alpha$ (positive constant)
$com = L/2$

* COM of half ring = $\frac{2R}{\pi}$



MR* COM O thi R/2 thi vadhare hovu jove mass ni najik O ane R thi nanu hovu jove

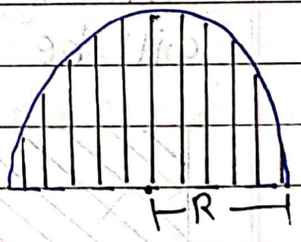
* COM of Circular Arc



$y_{COM} = \frac{R(\sin\theta/2)}{(\theta/2)}$

$(a_{avg})_c = \frac{v^2}{R}\frac{\sin\theta/2}{\theta/2}$ - Recap

* COM of Half Disc = $\frac{4R}{3\pi}$



* COM of Hollow Hemisphere = $\frac{4R}{8}$

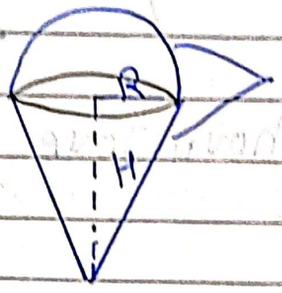
* COM of Solid Hemisphere = $\frac{3R}{8}$

} MR* COM Mass Dharavti Surface ni najik Ave chhe

* COM of Hollow cone = H/3 - Mass ni najik thodu upar

* COM of Solid cone = H/4 - Mass ni najik Niche

Q) COM of solid cone and solid Hemisphere is at centre then find relation bet. R and H



ρ = Same → mass per unit vol. same

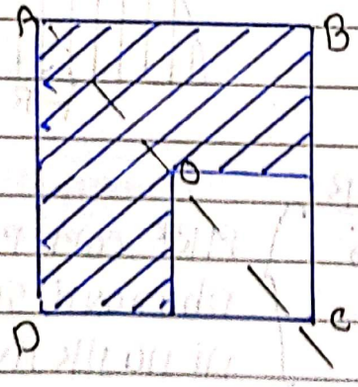
$$m_1 \frac{3R}{8} = m_2 \frac{H}{4}$$

$$\rho \frac{2}{3}\pi R^3 \frac{3R}{8} = \rho \frac{1}{3}\pi R^2 H \cdot \frac{H}{4}$$

$$3R^2 = H^2$$

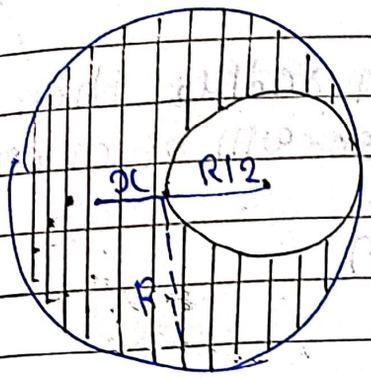
$$H = \sqrt{3}R$$

Q) From a uniform square plate, 1/4th part is removed As shown. The COM of remaining part will be lie on



OA

Q) From a uniform circular disc of mass M and radius R a small circular disc of radius R/2 is removed in such a way that both have a common tangent. Find the dist. of COM from the center of original disc



remaining part
R, 3M/4

If we put remaining part back into remaining part then COM of system should be lies at centre.

$$M_1R_1 = M_2R_2$$

$$\frac{3M}{4}x = \frac{M}{4}\cdot\frac{R}{2}$$

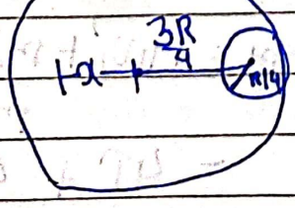
$$x = \frac{R}{6}$$

Q) If Disc of radius R/4 is removed from a uniform disc of mass M and radius R. then find location of COM of remaining part.

Remaining Part
R, $\frac{15M}{16}$

Removed part
$\frac{R}{4}$ $\frac{M}{16}$

$\frac{\pi R^2}{16}$

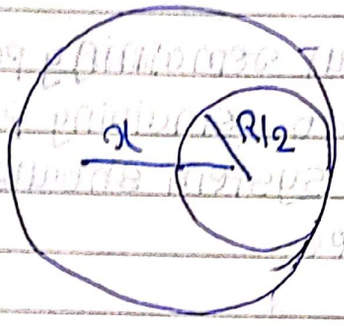


$$\left(\frac{15M}{16}\right)(x) = \left(\frac{M}{16}\right)\left(\frac{3R}{4}\right)$$

$$x = R/20$$

→ If Disc is cut coincide with center then COM of remaining part also will be at center

(Q) Find COM Remaining Position?



= Solid Sphere

$$\frac{M}{8} \times \frac{R}{2} = x \times \frac{7M}{8}$$

$$x = \frac{R}{14}$$

* Motion of COM

$$V_{COM} = \frac{m_1\vec{v_1} + m_2\vec{v_2}}{m_1 + m_2}$$

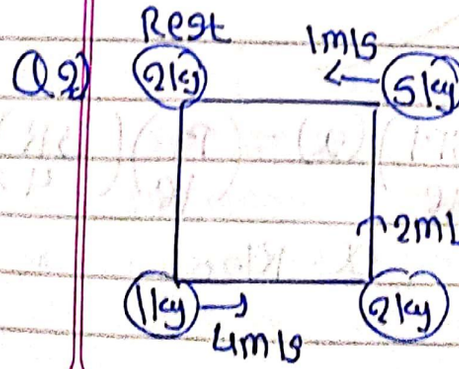
Q.1) (2kg) → 4m/s (4kg) → 2m/s

$$\vec{V}_{COM} = \frac{(2)(4) + (4)(2)}{6}$$

$$= \frac{8+8}{6} = \frac{14}{6}$$

$$\vec{V}_{COM} = 7\hat{i}$$

Q.2)



$$V_{COM} = \frac{m_1\vec{v_1} + m_2\vec{v_2} + m_3\vec{v_3} + m_4\vec{v_4}}{m_T}$$

$$= \frac{4\hat{i} + 4\hat{j} - 5\hat{i} + 0}{10}$$

$$= \left(-\frac{1}{10}\hat{i} + \frac{2}{5}\hat{j}\right) m/s$$

Q3) A ball of mass m is thrown upward and another ball is thrown downward, so as to move freely under gravity. The acc. of com

$$a_{com} = \frac{m_1\vec{a_1} + m_2\vec{a_2}}{m_1+m_2} = \frac{-g-g}{2} = -g \text{ m/s}^2$$

	A	B	C	D
1	○	○	○	○
2	○	○	○	○
3	○	○	○	○
4	○	○	○	○
5	○	○	○	○
6	○	○	○	○
7	○	○	○	○
8	○	○	○	○
9	○	○	○	○
10	○	○	○	○
11	○	○	○	○
12	○	○	○	○
13	○	○	○	○
14	○	○	○	○
15	○	○	○	○
16	○	○	○	○
17	○	○	○	○
18	○	○	○	○
19	○	○	○	○

	A	B	C	D
31	○	○	○	○
32	○	○	○	○
33	○	○	○	○
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35	○	○	○	○
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43	○	○	○	○
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45	○	○	○	○
46	○	○	○	○
47	○	○	○	○
48	○	○	○	○
49	○	○	○	○

	A	B	C	D
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62	○	○	○	○
63	○	○	○	○
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77	○	○	○	○
78	○	○	○	○
79	○	○	○	○

	A	B	C	D
91	○	○	○	○
92	○	○	○	○
93	○	○	○	○
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107	○	○	○	○
108	○	○	○	○
109	○	○	○	○

	A	B	C	D
121	○	○	○	○
122	○	○	○	○
123	○	○	○	○
124	○	○	○	○
125	○	○	○	○
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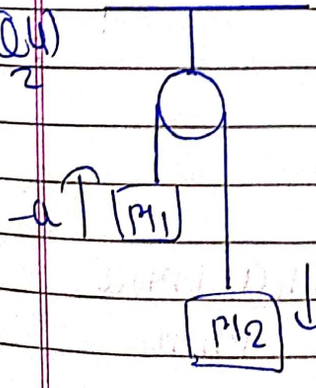
132	○	○	○	○
133	○	○	○	○
134	○	○	○	○
135	○	○	○	○
136	○	○	○	○
137	○	○	○	○
138	○	○	○	○
139	○	○	○	○

	A	B	C	D
151	○	○	○	○
152	○	○	○	○
153	○	○	○	○
154	○	○	○	○
155	○	○	○	○
156	○	○	○	○
157	○	○	○	○
158	○	○	○	○
159	○	○	○	○
160	○	○	○	○
161	○	○	○	○
162	○	○	○	○
163	○	○	○	○
164	○	○	○	○
165	○	○	○	○
166	○	○	○	○
167	○	○	○	○
168	○	○	○	○
169	○	○	○	○
170	○	○	○	○

Q4) Find a_{com}



$$a = \left(\frac{m_2-m_1}{m_1+m_2}\right)g$$

$$a_{com} = \frac{m_1a_1 + m_2a_2}{m_1+m_2}$$

$$a_{com} = \frac{-m_1\left[\frac{m_2-m_1}{m_1+m_2}\right] + m_2\left[\frac{m_2-m_1}{m_1+m_2}\right]}{m_1+m_2}$$

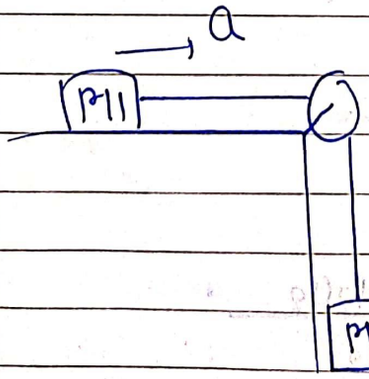
$$= \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g \left[\frac{m_2 - m_1}{m_1 + m_2}\right]$$

$$a_{com} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)^2 g$$

54	○ ● ○ ○	84	○ ○ ○ ○	114	○ ○ ○ ○	144	○ ○ ○ ○	174	○ ○ ○ ○
55	○ ● ○ ○	85	○ ○ ○ ○	115	○ ○ ○ ○	145	○ ○ ○ ○	175	○ ○ ○ ○
56	○ ○ ● ○	86	○ ○ ○ ○	116	○ ○ ○ ○	146	○ ○ ○ ○	176	○ ○ ○ ○
57	○ ○ ● ○	87	○ ○ ○ ○	117	○ ○ ○ ○	147	○ ○ ○ ○	177	○ ○ ○ ○
58	○ ○ ● ○	88	○ ○ ○ ○	118	○ ○ ○ ○	148	○ ○ ○ ○	178	○ ○ ○ ○
59	○ ● ○ ○	89	○ ○ ○ ○	119	○ ○ ○ ○	149	○ ○ ○ ○	179	○ ○ ○ ○
60	○ ○ ● ○	90	○ ○ ○ ○	120	○ ○ ○ ○	150	○ ○ ○ ○	180	○ ○ ○ ○

C D	A B C D	A B C D	A B C D	A B C D	A B C D
	31 ○ ○ ○ ○	61 ○ ○ ○ ○	91 ○ ○ ○ ○	121 ○ ○ ○ ○	151 ○ ○ ○ ○
	32 ○ ○ ○ ○	62 ○ ○ ○ ○	92 ○ ○ ○ ○	122 ○ ○ ○ ○	152 ○ ○ ○ ○
	33 ○ ○ ○ ○	63 ○ ○ ○ ○	93 ○ ○ ○ ○	123 ○ ○ ○ ○	153 ○ ○ ○ ○
	34 ○ ○ ○ ○	64 ○ ○ ○ ○	94 ○ ○ ○ ○	124 ○ ○ ○ ○	154 ○ ○ ○ ○
	35 ○ ○ ○ ○	65 ○ ○ ○ ○	95 ○ ○ ○ ○	125 ○ ○ ○ ○	155 ○ ○ ○ ○
	36 ○ ○ ○ ○	66 ○ ○ ○ ○	96 ○ ○ ○ ○	126 ○ ○ ○ ○	156 ○ ○ ○ ○
	37 ○ ○ ○ ○	67 ○ ○ ○ ○	97 ○ ○ ○ ○	127 ○ ○ ○ ○	157 ○ ○ ○ ○
	38 ○ ○ ○ ○	68 ○ ○ ○ ○	98 ○ ○ ○ ○	128 ○ ○ ○ ○	158 ○ ○ ○ ○
	39 ○ ○ ○ ○	69 ○ ○ ○ ○	99 ○ ○ ○ ○	129 ○ ○ ○ ○	159 ○ ○ ○ ○
	40 ○ ○ ○ ○	70 ○ ○ ○ ○	100 ○ ○ ○ ○	130 ○ ○ ○ ○	160 ○ ○ ○ ○
	41 ○ ○ ○ ○	71 ○ ○ ○ ○	101 ○ ○ ○ ○	131 ○ ○ ○ ○	161 ○ ○ ○ ○
	42 ○ ○ ○ ○	72 ○ ○ ○ ○	102 ○ ○ ○ ○	132 ○ ○ ○ ○	162 ○ ○ ○ ○
					163 ○ ○ ○ ○

Q.5)



$$a = \frac{m_2 g}{m_1 + m_2}$$

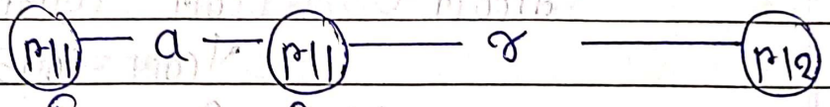
$$a_{com} = \frac{m_1 a_1 + m_2 a}{m_1 + m_2}$$

$$a_{com} = \frac{m_1 \left[\frac{m_2 g}{m_1 + m_2}\right] \hat{i} + m_2 \left[\frac{m_2 g}{m_1 + m_2}\right] \hat{j}}{m_1 + m_2}$$

$$a_{com} = \frac{m_1 m_2 g}{(m_1+m_2)^2}\hat{i} + \frac{m_2^2 g}{(m_1+m_2)^2}\hat{j}$$

* Shift in com

→ Two object of mass m_1 and m_2 is placed at dist. a. If mass m_1 is shifted left by dist. a as shown in figure. then find shift in com



Final Initial

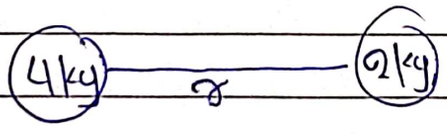
$$\vec{r}_{com} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}$$

$$\Delta\vec{r}_{com} = \frac{m_1\Delta\vec{r}_1 + m_2\Delta\vec{r}_2}{m_1+m_2}$$

↓

Shift in com

$$\text{Shift in com} = \frac{m_1 a}{m_1+m_2} \quad \text{m1R*}$$

Q) 

If 4kg mass is displaced by 5m left, then find shift in 2kg object so that com does not shift their position

$$\Delta x_{com} = \frac{m_1\Delta\vec{r}_1 + m_2\Delta\vec{r}_2}{m_1+m_2} \quad \therefore \quad 0 = (4)(-5) + (2)(\Delta r_2)$$

$$\Delta r_2 = 10m$$

Conservation of Momentum of COM

$$F_{ext} = \frac{d\vec{P}_{com}}{dt}$$

→ If external force acting on the particle is zero then momentum of com remain same

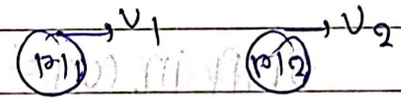
$F_{ext} = 0$

$dP_{com} = 0 \Rightarrow \vec{P}_{com} = \text{constant}$

$v_{com} = \text{constant}$

$a_{com} = 0$

→ If net force acting on the particle is zero, then momentum of a system remain constant. But can't say about K.E.



$F_{ext} = 0$

$\Delta P = 0$

$P_f = 0$ $\qquad$ $P_f = 0$

$\vec{P_1} + \vec{P_2} = 0$

$\vec{P_1} = -\vec{P_2}$

$KE_f = 0$ $\qquad$ $KE_f = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

Internal force can't change momentum but can change K.E

$Q_{\text{Internal force}} = \Delta KE = KEf = Q$ value of explosion $= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

Q1) Two particle A and B at rest initially move towards each other under a mutual force of Attraction. At the instant when velocity of A is v and that of B is v, the velocity of COM of system

$F_{ext} = 0$

$\Delta P = 0$ $\quad V_{COM} = \text{constant}$

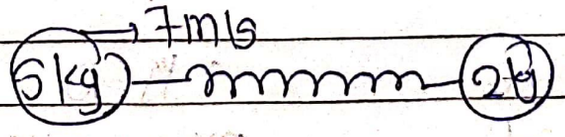
$V_1 = V_2 = 0$

$V_{COM} = 0$

Q2) A shell following a parabolic path explodes somewhere in its flight. The COM of fragment will move in
→ Parabolic path

Q3) A man of mass m is suspended in air by holding the rope a of a balloon of mass M. As the man climb up the rope, the balloon
→ Moves downward

Q4) Two block of mass 5kg and 2kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse provided a velocity 7m/s to the heavier block in the direction of lighter block. The velocity of COM is



$F_{ext} = 0$

$$V_{com} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{5 \times 7 + 0}{2+5}$$

$$V_{com} = \frac{5 \times 7}{7}$$

$$V_{com} = 5 \text{ m/s}$$

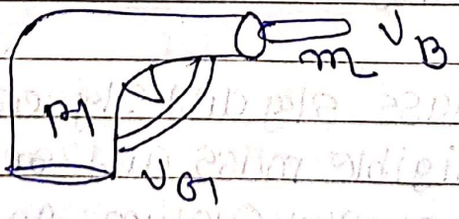
Q5) A body falling vertically downward under gravity breaks in two parts of unequal masses. The com of two parts together.

$(F_{ext})_{horizontal} = 0$

Don't shift at horizontal

Q6) A body at rest explodes into piece of mass unequal mass. The part will move in
→ opposite direction with unequal speed

* Gun-Bullet system



Initially at rest $P_i = 0$

finally $P_f = MV_G + mv_B$

$F_{ext} = 0$

$\Delta P = 0$

$P_f - P_i = 0$

$\therefore P_f = 0$

$MV_G + mv_B = 0$

$m_1 v_{G_1} = -m_2 v_B$

$v_{G_1} = -\frac{m_2 v_B}{m_1}$

$KE_P = 0$ $\quad KE_B = \frac{1}{2} m v_B^2$ $\quad KE_{G_1} = \frac{1}{2} m_1 v_{G_1}^2$

KE finally,

$KE_B = \frac{p^2}{2m}$ $\quad KE_{G_1} = \frac{p^2}{2m_1}$

$\therefore KE_B > KE_{G_1}$

Q7) A child is standing at one end of a long trolley moving with a speed v on a smooth horizontal track. If the child starts running towards the other end of the trolley with a speed v, the com of the system will move with a speed

→ v

Q8) A stationary particle explodes into two particle of mass m_1 & m_2 which move in opposite direction with vel. v_1 & v_2. The ratio of E_1/E_2 is

→ $\frac{E_1}{E_2} = \frac{m_2}{m_1}$

Q9) A shell of mass 200 gm is ejected from a gun of mass 4kg by an explosion that generate 1.05kJ of energy. The initial velocity of shell is

$K_{FG} + E_{shell} = 1050\ J$

$\frac{p^2}{2m_G} + \frac{p^2}{2m_S} = 1050$

$\frac{p^2}{2}\left[\frac{1}{0.2} + \frac{1}{4}\right] = 1050$

$p^2\left[5 + \frac{1}{4}\right] = 2100$

$\left[\frac{21}{4}\right]p^2 = 2100$

$p^2 = 400$

$\frac{u^2}{25} = 400$

$u = 20 \times 5 = 100\ m/s$

$p = mv$

$p = u(0.2)$

$p = \frac{1 \times u}{5}$

Q10) A body of mass (4m) is lying in xy plane at rest. It suddenly explodes into 3 pieces. Two pieces each of mass (m) move ⊥ to each other with equal speed (v). The total energy generated due to explosion is

$P_i = 0$

$P_f = 0$

$P_f = mv\hat{i} + mv\hat{j} + 2mv_0$

$0 = mv\hat{i} + mv\hat{j} + 2mv_0$

$2mv_0 = \sqrt{m^2v^2 + m^2v^2}$

$v_0 = \frac{\sqrt{2}mv}{2}$

$mv_0 = \frac{mv}{\sqrt{2}} \Rightarrow v_0 = \frac{v}{\sqrt{2}}$

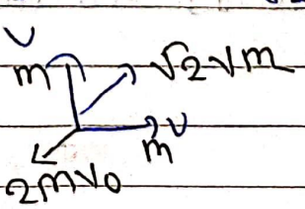
Total energy

$= E_1 + E_2 + E_3$

$= \frac{1}{2}\left[mv^2 + mv^2 + 2m\left(\frac{v}{\sqrt{2}}\right)^2\right]$

$= \frac{3mv^2}{2}$

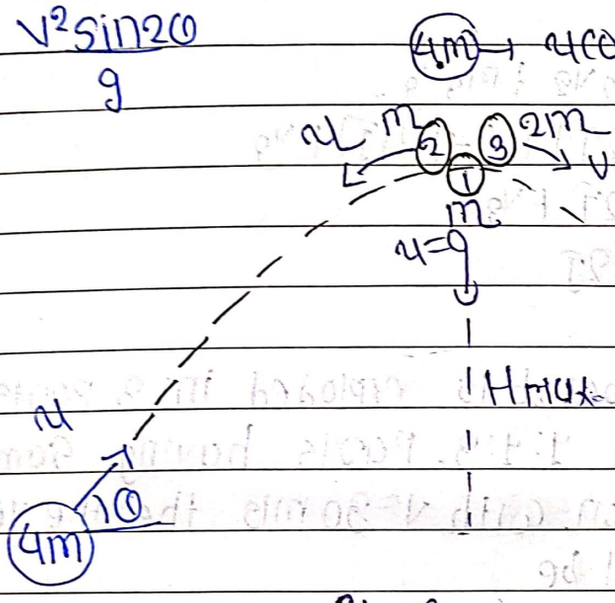
P1R*



Q(11) A bomb of mass m is projected from the ground with speed v at an angle θ with the horizontal. At the max. Height from the ground it explodes into two fragment of equal mass. If one fragment comes to rest immediately after explosion, then the horizontal range of com is

– $\frac{v^2 \sin 2\theta}{g}$

Q(12)



Ball 2 return via same path to the point of projection so that its speed will be $-u\cos\theta$

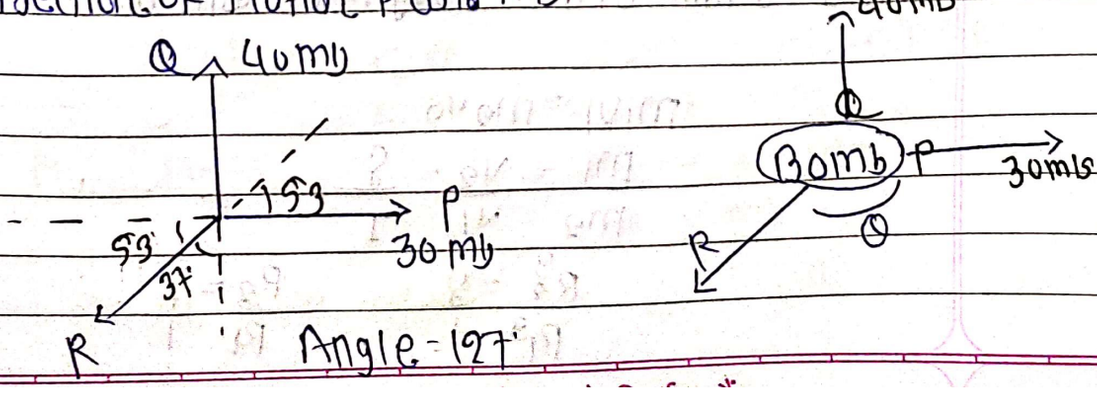
$P_i = P_f$

$4mu\cos\theta = 0 - mu\cos\theta + 2mv$

$5mu\cos\theta = 2mv$

$v = \frac{5u\cos\theta}{2}$

Q(13) A 6kg bomb at rest explodes into 3 equal pices. P, Q, R. If P flies with speed 30m/s and Q with speed 40m/s making an angle 90° with the direction of P. The angle bet. direction of motion P and R is.



Q14) A bomb Initially at rest explodes by itself into equal mass fragment. The vel of two fragment are $(3\hat{i}+2\hat{j})$ m/s and $(-\hat{i}-4\hat{j})$ m/s. Vel. of 3rd fragment will be

$P_i = P_f = 0$

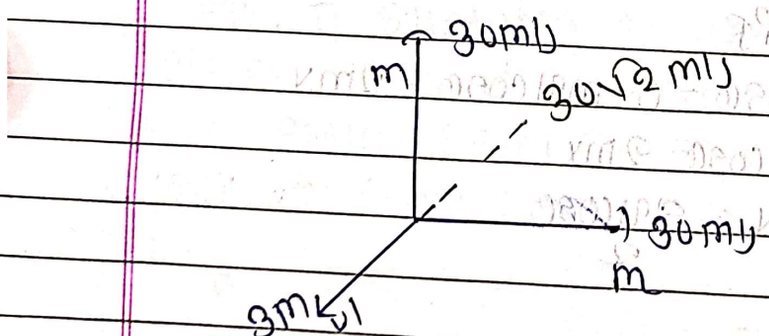
$P_f = m_1v_1 + m_2v_2 + m_3v_3$

$0 = (3\hat{i}+2\hat{j}) + (-\hat{i}-4\hat{j}) + v_3$

$\therefore \quad 0 = 2\hat{i} - 2\hat{j} + v_3$

$v_3 = -2\hat{i} + 2\hat{j}$

Q15) A 1kg stationary bomb is exploded in 3 parts having mass ratio 1:1:3. Parts having same mass move in ⊥ direction with v = 30 m/s then the vel. of biggest part will be



$3m\,v_1 = 30\sqrt{2}\,m$

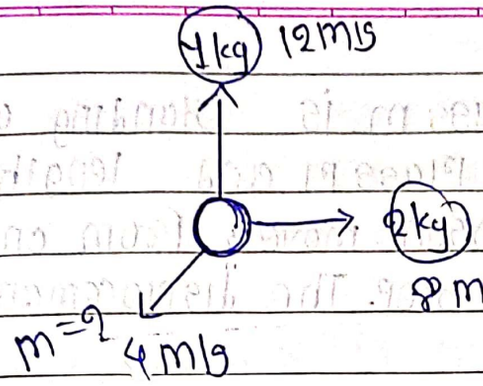
$v_1 = 10\sqrt{2}$ m/s

Q16) A heavy nucleus At rest explodes into two fragment which fly off with vel 8:1. The ratio of the radii of the fragment will be

$m_1v_1 = m_2v_2$

$\frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{8}{1}$

$\frac{R_2^3}{R_1^3} = \frac{8}{1} \qquad \frac{R_2}{R_1} = \frac{2}{1}$



$P_i = 0$

$P_f = 0$

$P_f = 0$

$m_1v_1 + m_2v_2 + m_3v_3 = 0$

$12\hat{i} + 16\hat{i} + 4m = 0$

$4m = -12\hat{i} + 16\hat{i}$

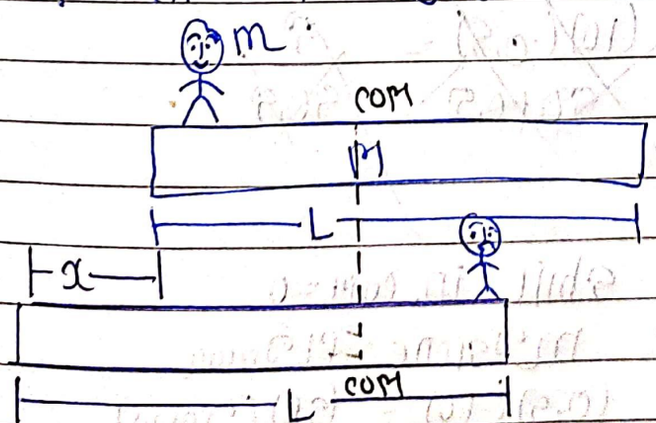
$4m = \sqrt{400}$

$m = \frac{20}{4}$

$m = 5\,kg$

* Man + Plank system

The net external force on the system (Man + Plank) is zero. The COM of the system is to be at the same position during their movement. But Plank shift left if men walk right.



$MR + m_1R = m_2R$

$\Delta r_{COM} = \frac{m_1\Delta\vec{r}_1 + m_2\Delta\vec{r}_2}{m_1+m_2}$

$m_1\Delta\vec{r}_1 + m_2\Delta\vec{r}_2 = 0$

$m(L-x) + M(-x) = 0$

$m(L-x) = Mx$

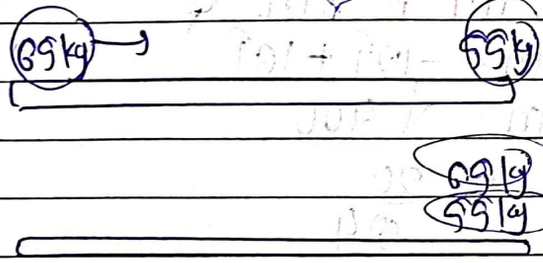
$mL - mx = Mx$

$x = \frac{mL}{M+m}$

Q1) A person of mass m is standing on one of the plank of mass M and length L and floating in water. The person moves from one end to another end and stop. The displacement of plank will be

$$x = \frac{mL}{M+m}$$

Q2)



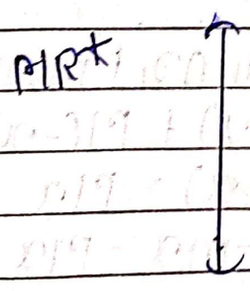
then com shift by

Ans. = 0

Q3) A man 50kg mass is standing in a gravity free space at a height of 10m above the floor. He throw a stone of mass 0.9kg mass downward with a speed of 2m/s. when stone reaches the floor the dist bet. man & stone will be

$$x = \frac{M\cancel{x}L}{\cancel{M+m}} \rightarrow \frac{(10)(0.9)}{50+0.5} = \frac{9}{50.5}$$

$x =$



shift in com = 0

$m x_{stone} = M x_{man}$

$(0.9)(10) = (50)(x_{man})$

$x_{man} = 0.1m$

Total dist. = 10 + 0.1 = 10.1m

Q4) A man of mass 80kg stands on a plank of mass 40kg. The plank is lying on a smooth horizontal surface. Initially both are at rest. The man starts walking on the plank towards North end stop After moving a dist. of 8m on the plank then.

$m_1 v_1 = m_2 v_2$

$(80)(6) = (40) v_2$

$v_2 = 12 m$ towards South plank will move

Jab kuch na likha ho to men ground k respect me move karega

Q.18) A shell of mass m moving with velocity v breaks up suddenly into two ... The part having mass m/3 remain stationary then vel. of other block will be

$F_{ext} = 0$

V_{com} constant

$$V_{com} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$mv = 0 + \frac{2m}{3} v_2$$

$$v_2 = \frac{3}{2} v$$